

THERE MAY BE NO HAUSDORFF ULTRAFILTERS

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ABSTRACT. An ultrafilter U is Hausdorff if for any two functions $f, g \in \omega^\omega$, $f(U) = g(U)$ iff $f \restriction X = g \restriction X$ for some $X \in U$. We will show that it is consistent that there are no Hausdorff ultrafilters.

1. INTRODUCTION

For $f \in \omega^\omega$ and an ultrafilter U on ω define $f(U) = \{X \subseteq \omega : f^{-1}(X) \in U\}$. Let \mathbf{FtO} be the collection of all finite-to-one functions $f \in \omega^\omega$.

Definition 1. Let U be an ultrafilter on ω . We say that

- (1) U is Hausdorff if for any two functions $f, g \in \omega^\omega$, if $f(U) = g(U)$ then $f \restriction X = g \restriction X$ for some $X \in U$.
- (2) U is weakly Hausdorff if for any two functions $f, g \in \mathbf{FtO}$, if $f(U) = g(U)$ then $f \restriction X = g \restriction X$ for some $X \in U$.

It is easy to see that

- (1) Ramsey ultrafilters are Hausdorff.
- (2) q -points are weakly Hausdorff.
- (3) Weakly Hausdorff p -points are Hausdorff.

It is worth mentioning that the following appears as an exercise in [9].

Lemma 2. If $f(U) = U$ then there exists $X \in U$ such that $f(n) = n$ for $n \in X$.

Therefore, if U is not Hausdorff, then this is witnessed by two functions, both not one-to-one mod U .

The notion of a Hausdorff ultrafilters was reintroduced and studied by Mauro Di Nasso, Marco Forti and others in a sequence of papers ([8], [7], [10] and [6]) in context of topological extensions. They used the name Hausdorff because Hausdorff ultrafilters are precisely those ultrafilters whose ultrapowers equipped with the standard topology are Hausdorff topological spaces. They asked whether the existence of a Hausdorff ultrafilter can be proved in ZFC. We will show that, at least for ultrafilters on ω , the answer is negative. However such ultrafilters (with various extra properties) may be constructed under from additional set theoretical assumptions (see [7]).

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2. CONSTRUCTION OF THE MODEL

In this section we will show how to build a model where there are no Hausdorff ultrafilters modulo the proofs of theorems 4 and 6 below.

Definition 3. An ultrafilter U is strongly non-Hausdorff if for every $f \in \text{FtO}$, $f(U)$ is not weakly Hausdorff.

Theorem 4. Assume CH. There exists a strongly non-Hausdorff p -point.

Definition 5. [2], [3], [5]. Let NCF stand for the following statement:
for any ultrafilters U, V on ω there exists $h \in \text{FtO}$ such that $h(U) = h(V)$.

Theorem 6. There exists a proper forcing notion \mathcal{P} such that

- (1) If $\mathbf{V} \models \text{GCH}$ then $\mathbf{V}^{\mathcal{P}} \models 2^{\aleph_0} = \aleph_2$,
- (2) If U is a p -point then $\mathbf{V}^{\mathcal{P}} \models U$ generates a p -point.
- (3) If U is strongly non-Hausdorff filter then $\mathbf{V}^{\mathcal{P}} \models U$ generates a strongly non-Hausdorff filter.
- (4) $\mathbf{V}^{\mathcal{P}} \models \text{NCF}$.

Theorem 7. Suppose that $\mathbf{V} \models \text{GCH}$. Then in $\mathbf{V}^{\mathcal{P}}$ there are no weakly Hausdorff ultrafilters. In particular, there are no Hausdorff ultrafilters in this model.

Proof. Let U_0 be a strongly non-Hausdorff p -point in \mathbf{V} given by theorem 4. By theorem 6, U_0 generates a strongly non-Hausdorff p -point in $\mathbf{V}^{\mathcal{P}}$, and $\mathbf{V}^{\mathcal{P}}$ satisfies NCF. So suppose that U is an ultrafilter in $\mathbf{V}^{\mathcal{P}}$. By NCF there exists $h \in \text{FtO}$ such that $h(U) = h(U_0)$. Since U_0 is strongly non-Hausdorff in $\mathbf{V}^{\mathcal{P}}$ it follows that $h(U_0)$ is not Hausdorff. On the other hand if U was Hausdorff then the following lemma would imply that $h(U)$ is Hausdorff as well, a contradiction.

Lemma 8. If U is Hausdorff then $h(U)$ is also Hausdorff.

Proof. Let $f, g \in \text{FtO}$ be such that $f(h(U)) = g(h(U))$. It follows that there is $X \in U$ such that $f \circ h \upharpoonright X = g \circ h \upharpoonright X$. Thus $f \upharpoonright h[X] = g \upharpoonright h[X]$ and $h[X] \in h(U)$. \square

\square

3. A STRONGLY NON-HAUSDORFF ULTRAFILTER

Let $I \subset \omega$ be a finite set and let $\Delta = \{(n, n) : n \in \omega\}$. Denote by $[I]^2 = (I \times I) \setminus \Delta$. For a set $X \subseteq [I]^2$ define

$$\|X\|_I = \min \left\{ k : \exists \{A_i, B_i : i \leq k\} \forall i \leq k \ A_i \cap B_i = \emptyset \text{ and } X \subseteq \bigcup_{i \leq k} A_i \times B_i \right\}.$$

We will drop the subscript I if it is clear from the context what it is.

Lemma 9. (1) $\|[I]^2\|_I \rightarrow \infty$ as $|I| \rightarrow \infty$.
 (2) $\|X \cup Y\|_I \leq \|X\|_I + \|Y\|_I$,
 (3) if $Z \subseteq I$ and $X \subseteq [I]^2$, $\|X\|_I > 2$, then either $\|[Z]^2 \cap X\|_I \geq \|X\|_I / 2 - 1$
 or $\|[I \setminus Z]^2 \cap X\|_I \geq \|X\|_I / 2 - 1$.

Proof. If (1) fails then there is $k \in \omega$ and sets $\{A_j^n, B_j^n : n, j \leq k\}$ such that $A_j^n \cap B_j^n = \emptyset$ for $j \leq k$ and $[n]^2 = \bigcup_{j \leq k} A_j^n \times B_j^n$. By compactness we get sets

$\{A_j^\omega, B_j^\omega : j \leq k\}$ such that $A_j^\omega \cap B_j^\omega = \emptyset$ for $j \leq k$ and $[\omega]^2 = \bigcup_{j \leq k} A_j^\omega \times B_j^\omega$, which is not possible.

A more direct argument shows that the following strategy is optimal for covering $[I]^2$, when $|I|$ is a power of two. Write $I = I_0 \cup I_1$ of equal size and use $I_0 \times I_1$ and $I_1 \times I_0$ to cover part of $I \times I$. For the rest, that is $(I_0 \times I_0) \cup (I_1 \times I_1)$ apply the same strategy by writing $I_0 = I_{00} \cup I_{01}$ and $I_1 = I_{10} \cup I_{11}$. The procedure terminates when squares have size 2×2 , that is after $\log_2(|I|) - 1$ steps. At that time we have used $2 + 2 \cdot 2 + 2 \cdot 4 + \dots + 2 \times 2^{\log_2(|I|)-1} = 2 \cdot |I| - 2$ rectangles. For I of arbitrary size we get (by rounding down to the nearest power of two) that $\|[I]^2\|_I \geq |I| - 2$.

(2) is obvious.

(3) Note that

$$\begin{aligned} \|X\|_I &\leq \|([Z]^2 \cup [I \setminus Z]^2 \cup (Z \times (I \setminus Z)) \cup ((I \setminus Z) \times Z)) \cap X\|_I \leq \\ &\quad \|[Z]^2 \cap X\|_I + \|[I \setminus Z]^2 \cap X\|_I + 1 + 1. \end{aligned}$$

Thus

$$\|[Z]^2 \cap X\|_I + \|[I \setminus Z]^2 \cap X\|_I \geq \|X\|_I - 2.$$

□

For $I \in [\omega]^{<\omega}$ let $\pi_1, \pi_2 : [I]^2 \longrightarrow I$ be projections onto first and second coordinate respectively.

Lemma 10. *Suppose that $X \subseteq [I]^2$, and $\|X\|_I > 2$. Then $\pi_0(X) \cap \pi_1(X) \neq \emptyset$.*

Proof. Suppose that $\pi_0(X) = u$ and $\pi_1(X) = v$. If $u \cap v = \emptyset$ then $X \subseteq (u \times v) \cup (v \times u)$. Thus $\|X\|_I \leq 2$. □

Next we define functions $f^0, g^0 \in \text{FtO}$ that will witness that ultrafilter U_0 that we are about to construct is not Hausdorff.

Let $\{I_k, J_k : k \in \omega\}$ be two sequences of disjoint consecutive intervals such that for $k \in \omega$,

- (1) $\|[I_k]^2\|_{I_k} \geq 2^{2^k}$,
- (2) $|J_k| = |[I_k]^2|$.

Bijection implicit in (2) allows us to define projections $\pi_0^k, \pi_1^k : J_k \longrightarrow I_k$. Let $f^0 = \bigcup_k \pi_0^k$ and $g^0 = \bigcup_k \pi_1^k$. Note that $f^0(x) \neq g^0(x)$ for any $x \in J_k = [I_k]^2$, $k \in \omega$.

As a warm-up let us use these definitions to show the following:

Lemma 11. *Assume CH. There exists a p -point that is not weakly Hausdorff.*

Proof. We will need the following easy observation:

Lemma 12. *If $f, g \in \text{FtO}$ and U is an ultrafilter then the following conditions are equivalent:*

- (1) $f(U) \neq g(U)$,
- (2) $f[X] \cap g[X] = \emptyset$ for some $X \in U$. □

We will build an ultrafilter V_0 on the set $\bigcup_k [I_k]^2$ which we identified with ω . Let $\{Z_\alpha : \alpha < \omega_1\}$ be enumeration of $[\omega]^\omega$.

We will build by induction a sequence $\{X_\alpha : \alpha < \omega_1\}$ so that

- (1) $\forall \beta < \alpha \ X_\alpha \subseteq^* X_\beta$,

- (2) $X_{\alpha+1} \cap Z_\alpha = \emptyset$ or $X_{\alpha+1} \subseteq Z_\alpha$ for all α .
- (3) for every $\alpha < \omega_1$, $f^0[X_\alpha] \cap g^0[X_\alpha] \neq \emptyset$.
- (4) for every $\alpha < \omega_1$, $\limsup_k \|X_\alpha \cap J_k\|_{I_k} = \infty$.

Let $V_0 = \{X : \exists \alpha X_\alpha \subseteq^* X\}$. Note that the conditions (1) and (2) guarantee that V_0 is a p -point, and lemma 12 and (3) implies that $f^0(V_0) = g^0(V_0)$. Finally, (4) is the requirement that (by lemma 10) implies (3).

SUCCESSOR STEP. Suppose that X_α is given. Find a strictly increasing sequence $\{\ell_k : k \in \omega\}$ such that the set $A = \{k : \|X_\alpha \cap J_k\|_{I_k} = \ell_k\}$ is infinite. Let $A_0 = \{k : \|X_\alpha \cap Z_\alpha \cap J_k\|_{I_k} \geq \ell_k/2 - 1\}$ and $A_1 = \{k : \|(X_\alpha \setminus Z_\alpha) \cap J_k\|_{I_k} \geq \ell_k/2 - 1\}$. By lemma 9(3), one of these sets, say A_0 , is infinite. Let $X_{\alpha+1} = \bigcup_{k \in A_0} X_\alpha \cap Z_\alpha \cap J_k$. The other case is the same.

LIMIT STEP. Given $\{X_\beta : \beta < \alpha < \omega_1\}$ let $\{\beta_k : k \in \omega\}$ be an increasing sequence cofinal in α . By finite modifications we can assume that $X_{\beta_{k+1}} \subseteq X_{\beta_k}$ for all k . Build by recursion a strictly increasing sequence $\{u_k : k \in \omega\}$ such that

$$\forall k \forall j \leq k \exists i \in [u_k, u_{k+1}) \|X_{\beta_j} \cap J_i\|_{I_i} \geq k,$$

and let

$$X_\alpha = \bigcup_k \left(X_{\beta_k} \cap \bigcup_{i \in [u_k, u_{k+1})} J_i \right).$$

It is clear that X_α satisfies (1) and (4). \square

Observe that CH was only needed in the limit step. If we do not require that that U is a p -point then we have the following:

Theorem 13. *There exists an ultrafilter that is not weakly Hausdorff.*

Proof. As in lemma 11, we will build an ultrafilter on the set $\bigcup_k [I_k]^2$. Let

$$\mathcal{I} = \left\{ X \subseteq \bigcup_k [I_k]^2 : \limsup_k \|X \cap J_k\|_{I_k} < \infty \right\}.$$

Note that \mathcal{I} is an ideal, and let U be any ultrafilter orthogonal to \mathcal{I} . Functions f^0, g^0 witness that U is not Hausdorff. \square

PROOF OF THEOREM 4.

Now we are ready to construct a p -point ultrafilter U_0 whose all finite-to-one images are not weakly Hausdorff.

Let $\{h_\alpha, Z_\alpha : \alpha < \omega_1\}$ be enumeration of FtO and $[\omega]^\omega$ respectively. We will build by induction sequences $\{f^\alpha, g^\alpha : \alpha < \omega_1\}$, $\{X_\alpha : \alpha < \omega_1\}$ so that

- (1) $\forall \beta < \alpha X_\alpha \subseteq^* X_\beta$
- (2) $X_{\alpha+1} \cap Z_\alpha = \emptyset$ or $X_{\alpha+1} \subseteq Z_\alpha$ for all α .
- (3) for every α , $f^{\alpha+1}, g^{\alpha+1}$ witness that $h_\alpha(U_0)$ is not Hausdorff, where $U_0 = \{X \subseteq \omega : \exists \alpha X_\alpha \subseteq^* X\}$.
- (4) $\forall \beta \forall \alpha \geq \beta f^{\beta+1} \circ h_\beta[X_\alpha] \cap g^{\beta+1} \circ h_\beta[X_\alpha] \neq \emptyset$.

As before, (1) and (2) guarantee that U_0 is a p -point, and (3) implies that U_0 is strongly non-Hausdorff, and (4) is a specific form of (3). Note that at the limit stages we only have to preserve the induction hypothesis. At the successor step we will first define an auxiliary function $e_{\alpha+1}$, and put $f^{\alpha+1} = f^0 \circ e_{\alpha+1} : h_\alpha[X_\alpha] \rightarrow \omega$ and $g^{\alpha+1} = g^0 \circ e_{\alpha+1} : h_\alpha[X_\alpha] \rightarrow \omega$. In other words, $f^{\alpha+1}, g^{\alpha+1}$ are copies of f^0, g^0 on the image of X_α via $e_{\alpha+1} \circ h_\alpha$.

Therefore we need to clarify condition (3) by imposing conditions on e_α and specifying the induction hypothesis.

Definition 14. *Let us say that a finite set $Y \subseteq X_\alpha$ is a (n, β, α) -witness if there exists $k \in \omega$ such that $\|e_{\beta+1} \circ h_\beta[Y] \cap J_k\|_{I_k} \geq n$.*

To satisfy (3), we demand that for $\beta < \alpha < \omega_1$,

- (5) $\limsup_k \|e_{\beta+1} \circ h_\beta[X_\alpha] \cap J_k\|_{I_k} = \infty$, or equivalently
- (6) $\forall n \exists Y \in [X_\alpha]^{<\omega}$ Y is a (n, β, α) -witness.

LIMIT STEP.

Suppose that $\{X_\beta : \beta < \alpha\}$ are defined and α is a limit ordinal. Let $\{\beta_k : k \in \omega\}$ be an increasing sequence cofinal in α , and let $\{\gamma_k : k \in \omega\}$ be an enumeration of α such that $\gamma_j \leq \beta_k$ for $j \leq k$. Without loss of generality we can assume that $X_{\beta_n} \subseteq X_{\beta_m}$ for $n \geq m$.

Build by recursion a strictly increasing sequence $\{u_k : k \in \omega\}$ such that

$$\forall k \forall l, j \leq k \exists i \in [u_k, u_{k+1}) \quad \|e_{\gamma_l+1} \circ h_{\gamma_l}[X_{\beta_j}] \cap J_i\|_{I_i} \geq k,$$

and let

$$X_\alpha = \bigcup_k \left(X_{\beta_k} \cap \bigcup_{i \in [u_k, u_{k+1})} J_i \right).$$

It is clear that X_α satisfies (1) and (4).

SUCCESSOR STEP.

Suppose that X_α satisfying (4) is already defined and we want to define $X_{\alpha+1}$ and $e_{\alpha+1}$ satisfying (2) and (5). Recall that by the induction hypothesis, for $\beta < \alpha$,

$$\forall n \exists Y \in [X_\alpha]^{<\omega} \quad Y \text{ is a } (n, \beta, \alpha)\text{-witness.}$$

Let $\{\beta_k : k \in \omega\}$ be an enumeration of α . Find a sequence $\{E_k : k \in \omega\}$ of consecutive intervals such that

- (1) $\forall k \forall j \leq k \quad h_\alpha^{-1}(E_k)$ contains a (k, β_j, α) -witness.
- (2) $\forall k \quad E_k \cap h_\alpha[X_\alpha] \neq \emptyset$.

Let $e_{\alpha+1}(j) = k \iff j \in E_k$ for $j \in \omega$. Condition (2) implies that $e_{\alpha+1} \circ h_\alpha[X_\alpha] = \omega$. In particular, either $\limsup_k \|e_{\alpha+1} \circ h_\alpha[Z_\alpha \cap X_\alpha] \cap J_k\|_{I_k} = \infty$, or $\limsup_k \|e_{\alpha+1} \circ h_\alpha[X_\alpha \setminus Z_\alpha] \cap J_k\|_{I_k} = \infty$. Let $X_{\alpha+1}$ be the appropriate set. It remains to check that for $\beta \leq \alpha$, $\limsup_k \|e_{\beta+1} \circ h_\beta[X_{\alpha+1}] \cap J_k\|_{I_k} = \infty$. If $\beta = \alpha$ it follows immediately from the definition of $X_{\alpha+1}$, so suppose that $\beta = \beta_j < \alpha$.

Let $m \in e_{\alpha+1} \circ h_\alpha[X_{\alpha+1}] \setminus j$ and note that $(e_{\alpha+1} \circ h_\alpha)^{-1}(m)$ contains a (m, β_j, α) -witness. It follows that $\limsup_k \|e_{\beta+1} \circ h_\beta[X_{\alpha+1}] \cap J_k\|_{I_k} = \infty$, which finishes the construction.

4. FORCING

Since known models for NCF are obtained by countable support iteration we will look for a proper forcing notion \mathbb{P} such that the iteration of \mathbb{P} has the required properties.

Suppose that \mathbb{P} is a proper forcing notion. \mathbb{P} preserves non-meager sets if for every countable elementary submodel $N \prec \mathbf{H}(\chi)$ containing \mathbb{P} , a condition $p \in \mathbb{P} \in N$ and a Cohen real c over N there exists $q \geq p$ such that q is (N, \mathbb{P}) generic and $q \Vdash_{\mathbb{P}} c$ is Cohen over $N[\dot{G}]$.

By [1], 6.3.16 this is equivalent to the property $\sqsubseteq^{\text{Cohen}}$ defined in [1].

Let \mathbb{P} be a proper forcing notion such that:

- (1) \mathbb{P} preserves p -points,
- (2) \mathbb{P} preserves non-meager sets, that is it preserves $\sqsubseteq^{\text{Cohen}}$.

Let $\mathcal{P} = \mathbb{P}_{\omega_2}$ be the countable support iteration of \mathbb{P} of length ω_2 . We have the following:

- (1) \mathcal{P} preserves p -points (see [4] or [1] 6.2.6),
- (2) \mathcal{P} preserves non-meager sets ([1], 6.3.20).

Recall that if \mathbb{P} is either Blass-Shelah forcing from [4] or Miller superperfect forcing, then \mathcal{P} has the above properties (7.3.46 and 7.3.48 of [1]) and $\mathbf{V}^{\mathcal{P}} \models \text{NCF}$, [5] or [4]. Therefore the following theorem concludes the proof of theorem 6.

Theorem 15. *Suppose that \mathcal{P} is a proper forcing that preserves non-meager sets and $U_0 \in \mathbf{V}$ is a strongly non-Hausdorff ultrafilter defined earlier. Then*

$$\mathbf{V}^{\mathcal{P}} \models U_0 \text{ generates a strongly non-Hausdorff filter.}$$

Proof. Clearly, U_0 may not generate an ultrafilter in the extension, for example when \mathcal{P} is Cohen forcing.

Let \mathbf{C} be the Cohen forcing interpreted as adding a function $c \in \text{FtO}$. Specifically, the conditions are finite sequences of consecutive intervals $\{I_k : k < n\}$ and $c(i) = k \iff i \in I_k$.

Lemma 16. *Let c be Cohen reals over \mathbf{V} . Then for every $h \in \mathbf{V} \cap \text{FtO}$, $h(U_0)$ is not Hausdorff as witnessed by $f^h = f^0 \circ c$ and $g^h = g^0 \circ c$.*

Proof. This is quite easy. Given $s = \{L_k : k < n\} \in \mathbf{C}$, $X = X_\alpha \in U_0$ we extend s by adding an interval L_n so large that $L_n \supseteq (e_{\alpha+1})^{-1}(k)$ for some $k > n$. That means that $f^0 \circ c$ and $g^h = g^0 \circ c$ agree with $f^0 \circ e_{\alpha+1}$ and $g^0 \circ e_{\alpha+1}$ on long enough segments to witness that $f^0 \circ c[X] \cap g^0 \circ c[X] \neq \emptyset$. \square

Let \dot{h} be a \mathcal{P} -name for an element of FtO . Let $N \prec \mathbf{H}(\chi)$ be a countable submodel containing $U_0, \dot{h}, p, \mathcal{P}$ and let $c \in \mathbf{V} \cap \text{FtO}$ be a Cohen real over N . Since \mathcal{P} preserves non-meager sets there is $q \geq p$ which is (N, \mathcal{P}) generic and $q \Vdash_{\mathcal{P}} c$ is Cohen over $N[\dot{G}]$. In particular, by lemma 16,

$$q \Vdash_{\mathcal{P}} \dot{h}(U_0) \text{ is not Hausdorff as witnessed by } f^0 \circ c \text{ and } g^0 \circ c.$$

By elementarity, it means that $\mathbf{V}^{\mathcal{P}} \models h(U_0)$ is not Hausdorff. \square

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